The Bernoulli Equation

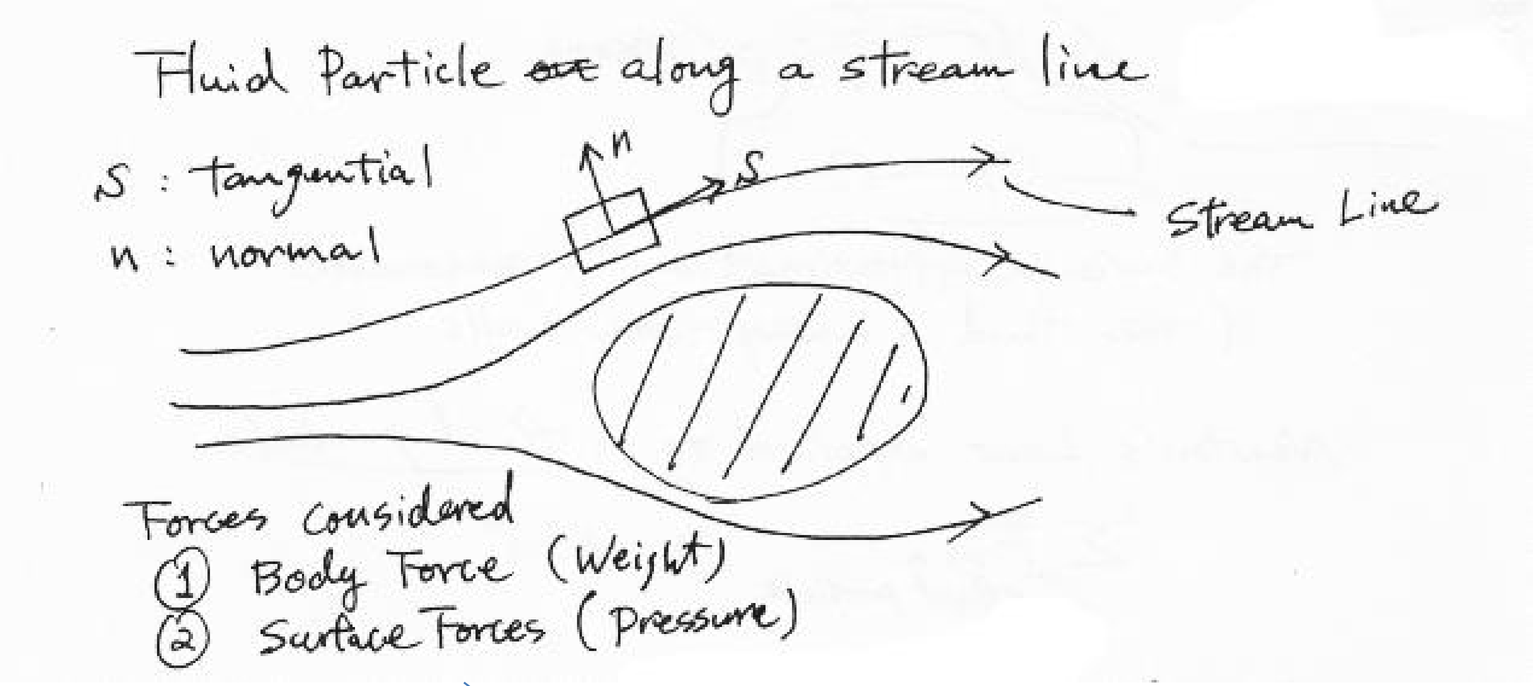
4 Assumptions (I I S S)

1) The flow is Inviscid
2) The flow is Incompressible
3) The flow Is Steady
4) We follow a particle along a Streamline

Those are 2 conflicting terms, no? No, because…of 3.

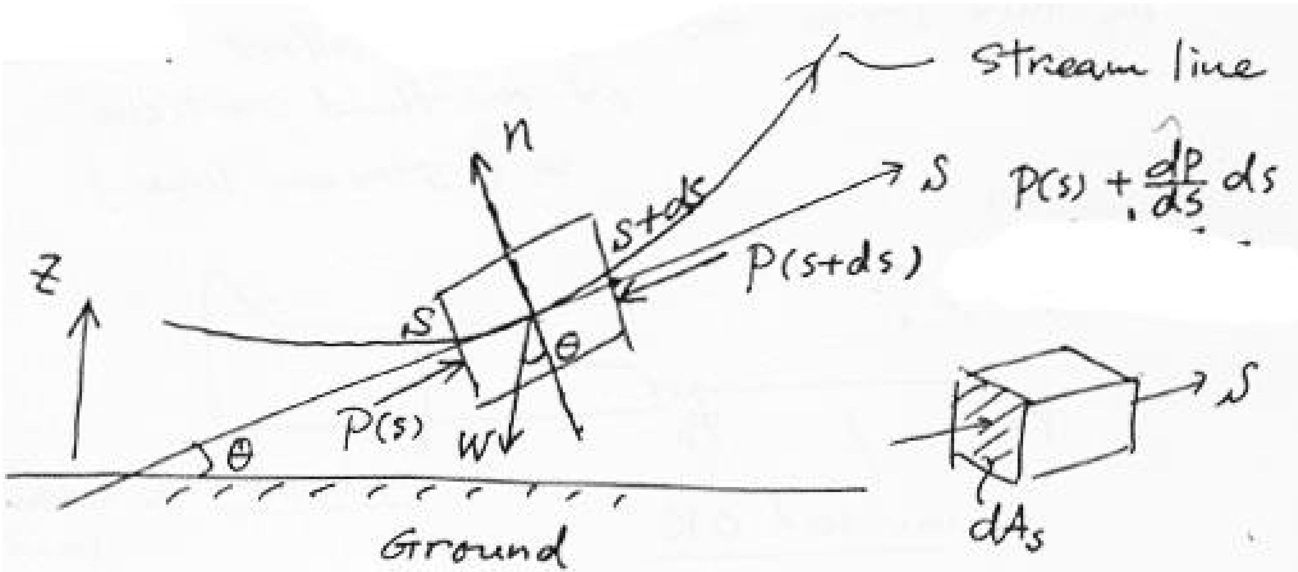
Can you mathematically describe those assumptions?

Lets follow a fluid particle along a streamline*



Why not shear stresses?

Lets draw the FBD.



Two points:

-We consider 'moving' coordinate system (s,n), where s is the position vector

- Do not forget that: s(t) and n(t) !!!

Apply Newton's Second Law

Newton's Law applied to a fluid particle

$$\sum \vec{F}_{\substack{\text{on a} \\ \text{fluid particle}}} = m\vec{a}$$

S-direction

Force along s-direction

$$\sum F_s = p(s)\,dA - p(s+ds)\,dA - W\sin\theta$$

$$= \cancel{p(s)dA} - [\cancel{p(s)} + \frac{dp}{ds}ds]\,dA - \gamma\, ds\, dA \sin\theta$$

$$= -\left[\frac{dp}{ds} + \gamma\sin\theta\right] dA\, ds$$

$$\sum F_s = -\left[\frac{dp}{ds} + \gamma\frac{dz}{ds}\right] dA\, ds$$

$$= \underbrace{\rho\, dA\, ds}_{\text{mass}}\ \underbrace{a_s}_{\text{Acceleration}}$$

How can you mathematically express acceleration, e.g in *s-direction?*

Velocity along the stream line $V = V(s)$

$$a_s = \frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} = \frac{dV}{ds}V$$

Because s(t)…

It should be V=V(s,t), no?

And finally…

$$\frac{d}{ds}\left[p + \frac{1}{2}\rho V^2 + \gamma z \right] = 0$$

Which means that:

$$\Rightarrow \boxed{p + \frac{1}{2}\rho V^2 + \gamma z = \text{const}} \text{ along a streamline}$$

This is the Bernoulli equation…very powerful tool….

In summary…

<u>Bernoulli Eqn:</u>

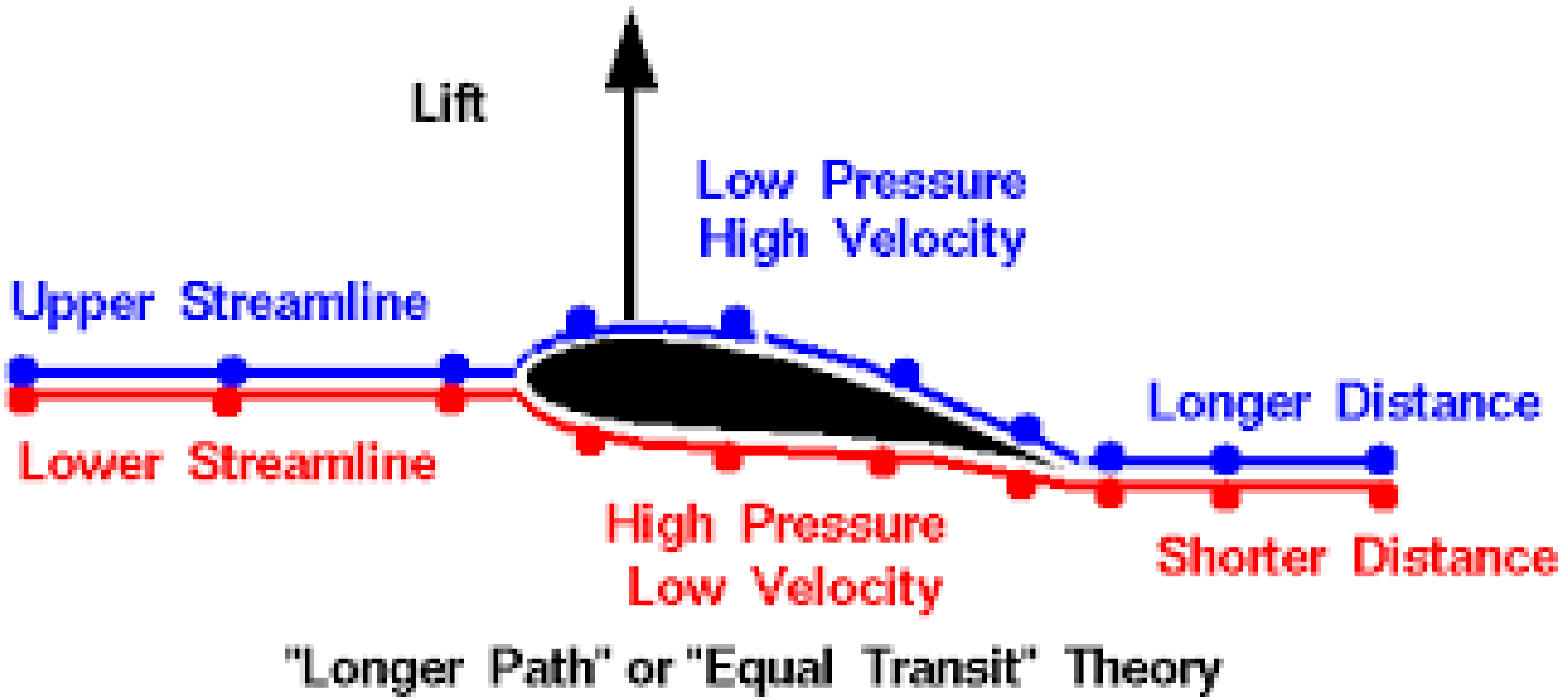
$\vec{F} = m\vec{a}$ applied to

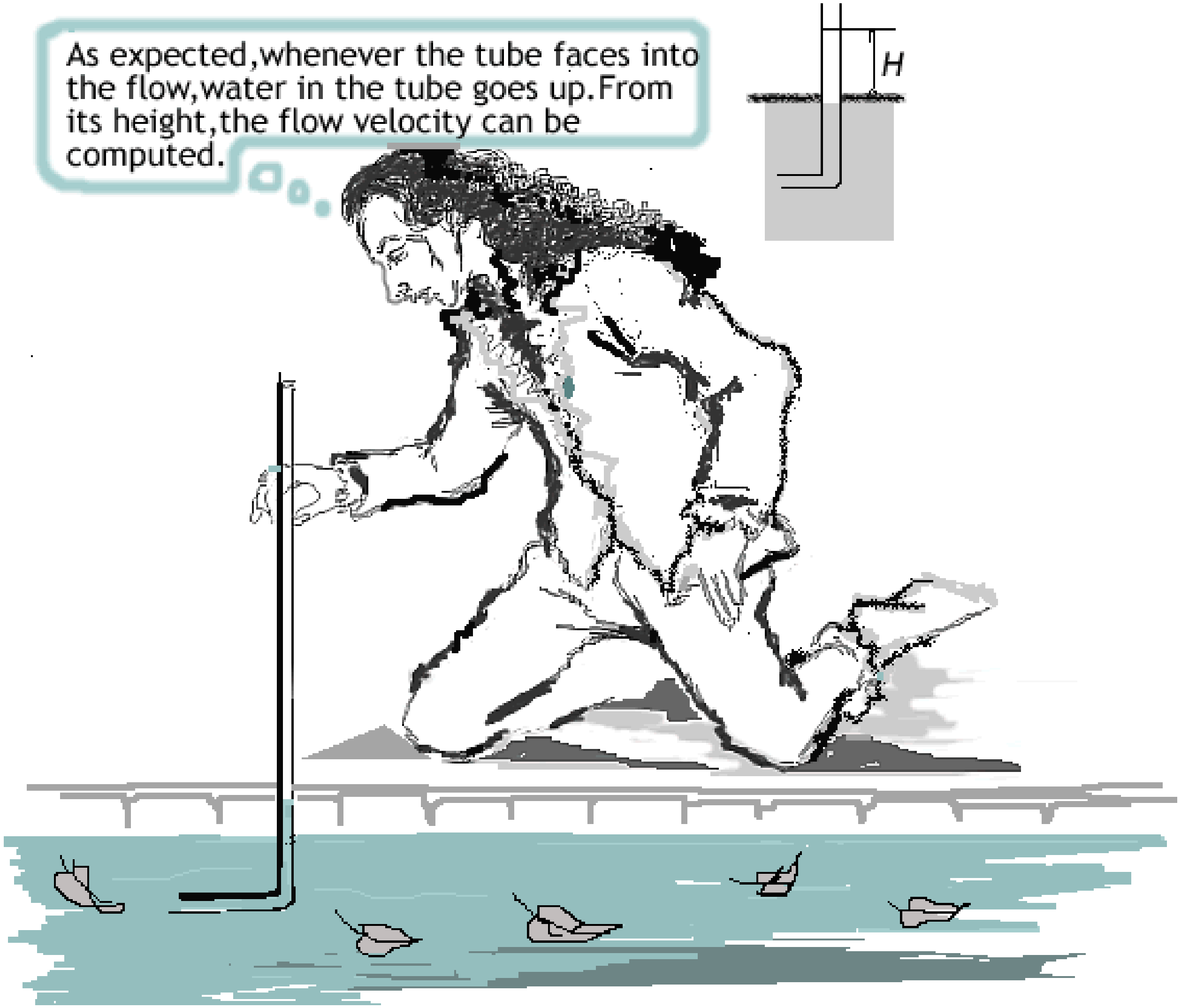
$\underset{I}{\underline{\text{inviscid}}}$, $\underset{I}{\underline{\text{incompressive}}}$, $\underset{S}{\underline{\text{steady}}}$ fluids

$$\left.\begin{array}{l} \boxed{-\dfrac{dp}{ds} - \gamma\dfrac{dz}{ds} = \rho V\dfrac{dV}{ds}} \\ \boxed{p + \tfrac{1}{2}\rho V^2 + \gamma z = \text{const}} \end{array}\right\} \text{along a stream line}$$

$$\left.\begin{array}{l} \boxed{-\dfrac{\partial p}{\partial n} - \gamma\dfrac{dz}{dn} = \rho\dfrac{V^2}{R}} \\ \boxed{p + \rho\displaystyle\int\frac{V^2}{R}dn + \gamma z = \text{const}} \end{array}\right\} \text{normal to a stream line}$$

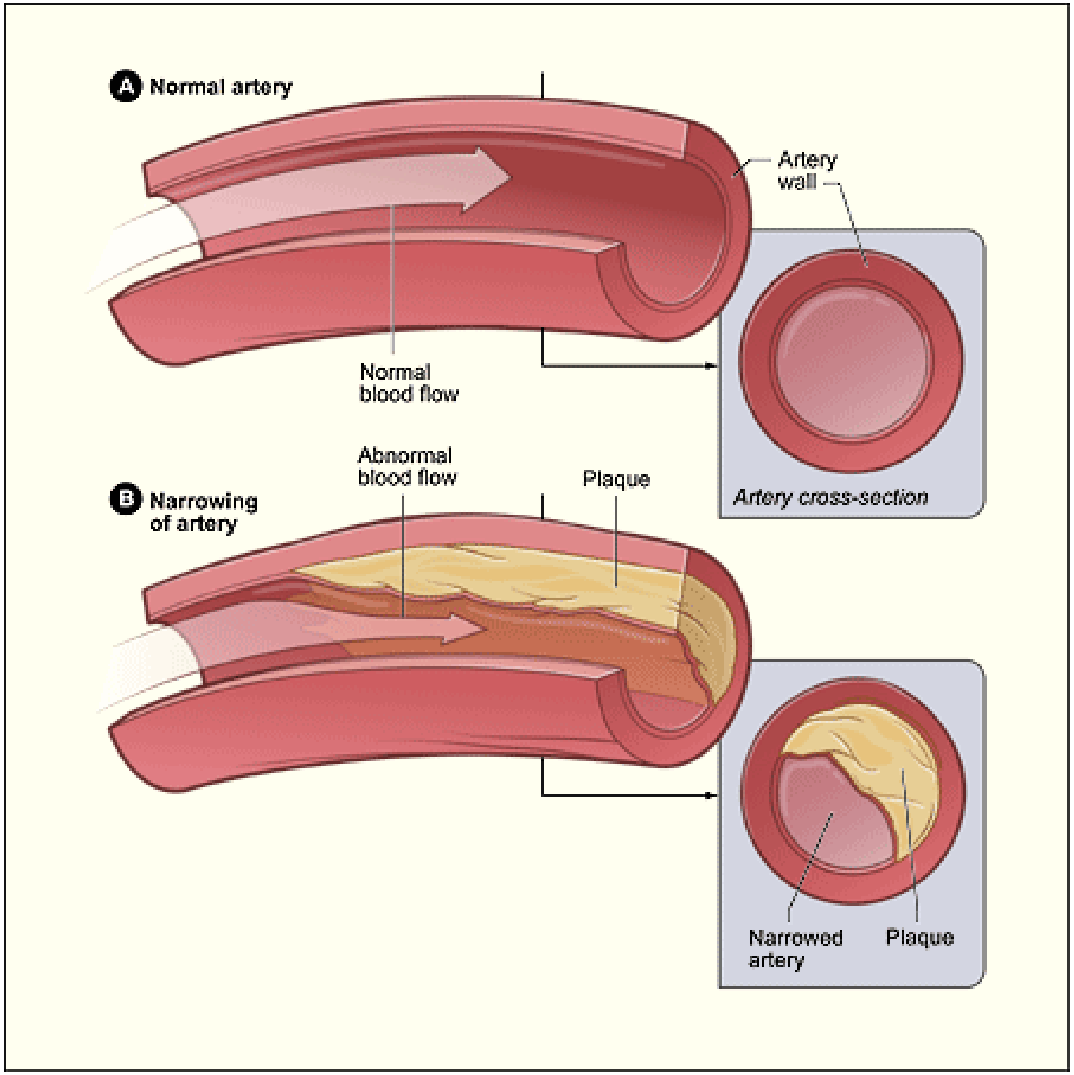
Applications: how airplanes fly?





http://nptel.ac.in/courses/112104118/15

Atherosclerosis



Plaque is mainly made up of fat, cholesterol and calcium

Today

- EXAMPLES OF BERNOULLI EQUATION (B.E.)

- And Unsteady B.E.

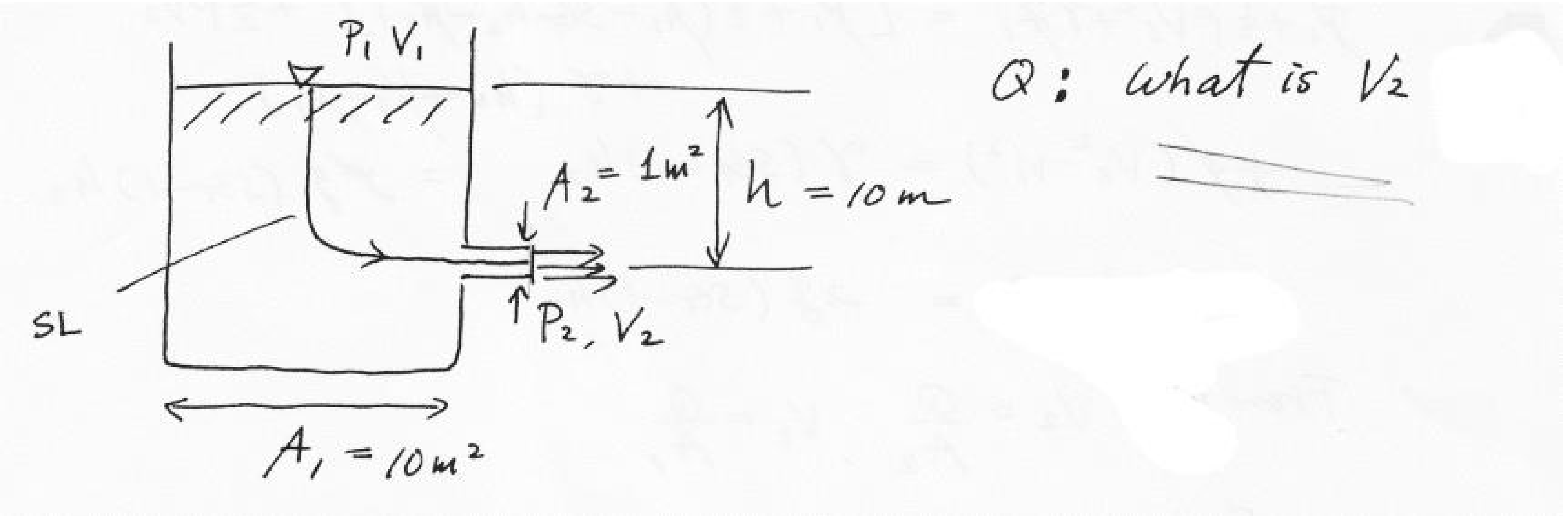
So Far…

Assuming : I I S S

· no viscous loss (no friction)
· no shaft work
· no heat transfer
· no chemical reaction.

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g Z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g Z_2 = constant$$

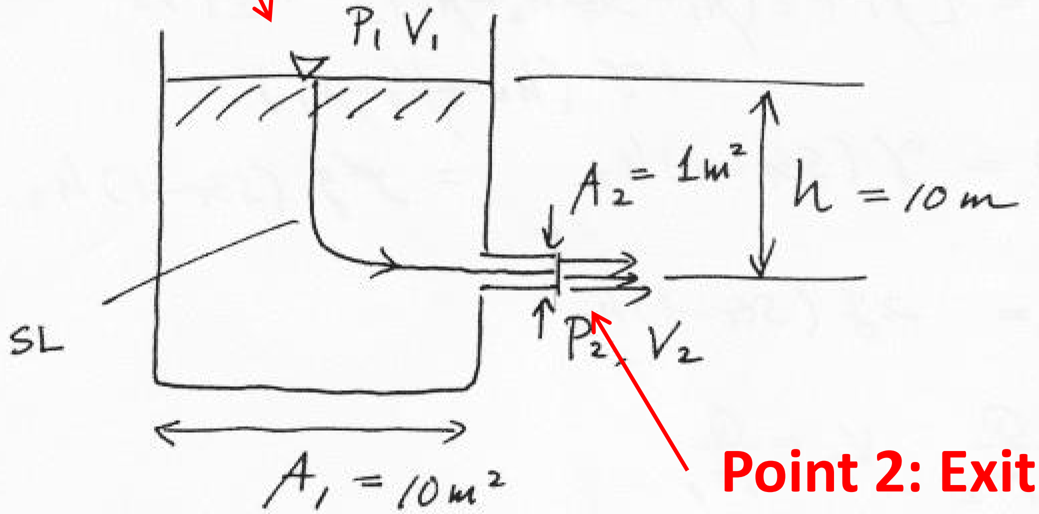
Example 1: Free Jet in a tank



<u>Step 1</u>: Visualize/draw Streamlines

<u>Step 2</u>: Identify Points of Interest on the Streamlines

Point 1: Free Surface



Point 2: Exit point

Q: what is V_2

Bernoulli Eqn.

$$\cancelto{0}{P_1} + \frac{1}{2}\rho V_1^2 + \gamma z_1 = \cancelto{0}{P_2} + \frac{1}{2}\rho V_2^2 + \cancelto{0}{\gamma z_2}$$

At free surfaces $P_1 = 0$, $P_2 = 0$

$z_1 = h$, $z_2 = 0$

Free Jet

$$\frac{1}{2}\cancel{\rho}(V_2^2 - V_1^2) = \gamma h = \cancel{\rho} g h$$

$$\boxed{V_2^2 - V_1^2 = 2gh}$$

We do not know V_1

Use Conservation of Mass:

$$V_2 = \frac{Q}{A_2} \qquad V_1 = \frac{Q}{A_1} \quad (\text{continuity})$$

$$Q^2\left[\left(\frac{1}{A_2}\right)^2 - \left(\frac{1}{A_1}\right)^2\right] = 2gh$$

$$Q = \sqrt{\frac{2gh}{\left(\frac{1}{A_2}\right)^2 - \left(\frac{1}{A_1}\right)^2}}$$

$$V_2 = \frac{Q}{A_2} = \frac{1}{A_2}\sqrt{\frac{2gh}{\left[\left(\frac{1}{A_2}\right)^2 - \left(\frac{1}{A_1}\right)^2\right.}} = \underline{\underline{14 \text{ m/s}}} \text{ Ans.}$$