1

Optical gain

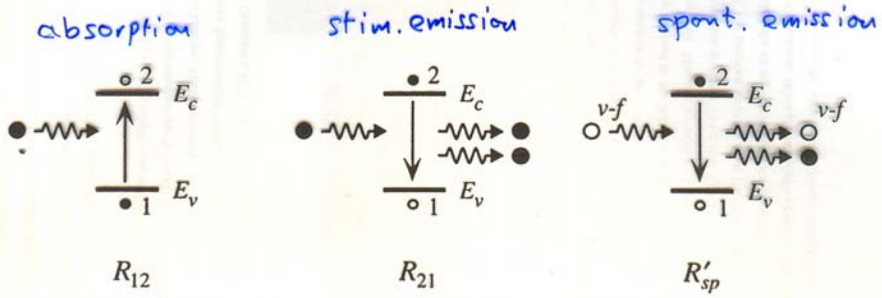

FIGURE 4.1 Band-to-band radiative transitions: stimulated absorption, stimulated emission, and spontaneous emission. (All rates are defined per unit volume.)

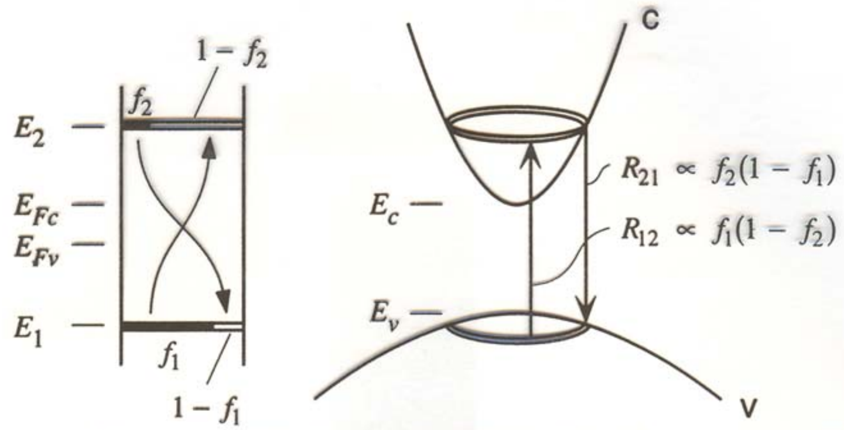

FIGURE 4.2 State pairs which interact with photons at E_{21}. Energy and momentum conservation reduce the set of state pairs to the annulus shown in the plot of energy vs. momentum in two dimensions. The occupation probabilities, f_1 and f_2, reduce this set even further.

- vertical transitions

- $$R_{12} = R_r \cdot f_1 \cdot (1-f_2)$$
  $$R_{21} = R_r \cdot f_2 \cdot (1-f_1)$$
  $$R_{sp}' = R_r^{v-f} \cdot f_2 \cdot (1-f_1)$$

  $R_r \propto |\mathcal{E}|^2$

  $R_r^{v-f} \propto |\mathcal{E}^{v-f}|^2$

- $$R_{st} \triangleq R_{21} - R_{12} = R_r \cdot (f_2 - f_1)$$

$$f_1 = \frac{1}{e^{(E_1 - E_{Fv})/kT} + 1}$$

$$f_2 = \frac{1}{e^{(E_2 - E_{Fc})/kT} + 1}$$

$$\frac{R_{21}}{R_{12}} = \frac{f_2(1-f_1)}{f_1(1-f_2)} = e^{(\Delta E_F - E_{21})/kT}$$

which implies for $R_{21} > R_{12}$ that

$$\boxed{\Delta E_F = E_{Fc} - E_{Fv} > E_{21} > E_g}$$

(this means also that the voltage that has to be applied in a diode laser $\Delta V > E_g$. For GaAs operating voltage $> 1.5V$. For GaN operating voltage $> 3.5V$)

$$\frac{R_{sp}'}{R_{st}} = \frac{|\varepsilon^{v-f}|^2 \cdot f_2(1-f_1)}{|\varepsilon|^2 (f_2 - f_1)} = \frac{|\varepsilon^{v-f}|^2}{|\varepsilon|^2} \cdot \frac{1}{1-e^{(E_{21}-\Delta E_F)/kT}}$$

(fundamental relation between spont. emission rate and net stimulation rate.)

Wavefunctions - Enveloppe function approximation

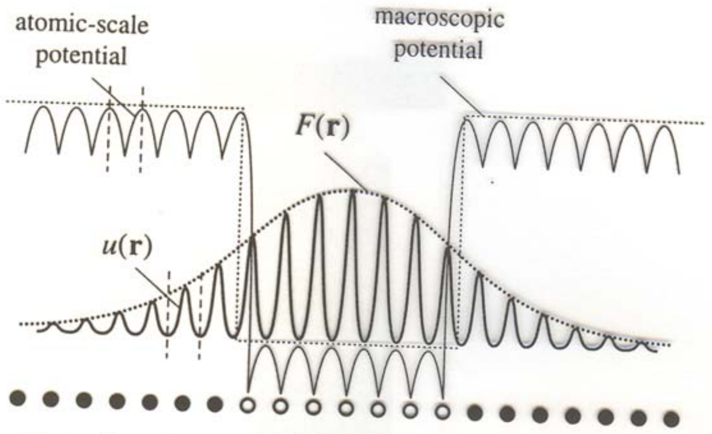

FIGURE 4.3 Illustration of a quantum-well potential and the corresponding lowest energy electron wavefunction.

$V \longrightarrow$ atomic-scale potential +

macroscopic potential slowly varying following the profile of the CB or VB edge.

(holes) $\Psi_1 = F_1(\vec{r}) \cdot u_v(\vec{r})$

(electrons) $\Psi_2 = F_2(\vec{r}) \cdot u_c(\vec{r})$

$\Big\}$ enveloppe function approximation

u_c, u_v = Block functions with crystal periodicity

F_1, F_2 = enveloppe functions.

bulk $\rightarrow$ $F(\vec{r}) = e^{-i\vec{k}\vec{r}} / \sqrt{V}$

2D $\rightarrow$ $F(\vec{r}) = F(z) \cdot e^{-i\vec{k}\vec{r}_{\parallel}} / \sqrt{A}$

1D $\rightarrow$ $F(\vec{r}) = F(x,y) \cdot e^{-ik_z \cdot z} / \sqrt{L}$

Time-dependent perturbation theory.

$$\vec{A}(\vec{r},t) = \hat{e}\ \mathrm{Re}\left\{ \tilde{A}(\vec{r})\cdot e^{i\omega t} \right\} =$$

$$= \hat{e}\cdot\frac{1}{2}\cdot\left[\tilde{A}(\vec{r})\cdot e^{i\omega t} + cc\right]$$

$$\left(\vec{\mathcal{E}} = -\frac{\partial \hat{A}}{\partial t}\right)$$

$$\vec{p}^{\,2} \to (\vec{p}+q\vec{A})^2 \simeq p^2 + \underbrace{2q\cdot\vec{A}\cdot\vec{p}}_{\text{dipole operator}}$$

$$H = H_0 + \left[H'(r)\cdot e^{i\omega t} + h.c \right]$$

where $\boxed{H'(r) = \frac{q}{2m_0}\cdot A(\vec{r})\cdot\hat{e}\cdot\vec{p}}$

Fermi's golden rule: $\boxed{R_r = \frac{2\pi}{\hbar}\cdot|H'_{21}|^2\cdot \underbrace{\rho_f(E_{21})\Big/_{E_{21}=\hbar\omega}}}$

equivalent to density of transition pairs per unit energy around $E_{21} = \hbar\omega$

$\left\{ \begin{array}{c} \text{reduced} \\ \text{density of} \\ \text{states} \end{array} \right\} \to \rho_r(E_{21})$

⑥

$$\frac{1}{\rho_r} = \frac{1}{\rho_c} + \frac{1}{\rho_v}$$

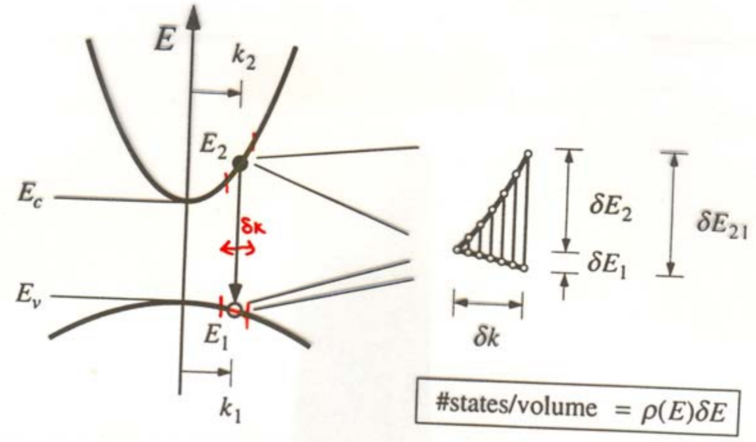

FIGURE 4.5 Relationship between the energy ranges in the conduction and valence bands for a given dk in k-space, assuming **k**-selection applies.

$$\left.\begin{array}{l}\rho_r \cdot \delta E_{21} = \rho_c \cdot \delta E_2 = \rho_v \cdot \delta E_1 \\ \text{since } \delta E_{21} = \delta E_1 + \delta E_2\end{array}\right\} \Rightarrow \frac{1}{\rho_r} = \frac{1}{\rho_c} + \frac{1}{\rho_v}$$

TABLE 4.3 Density of States for Bulk (3D), Quantum-Well (2D), and Quantum-Wire (1D) Structures (Including Spin).

Dimension	$\rho(k)$	$\rho(E)$
3	$\frac{k^2}{\pi^2}$	$\frac{\sqrt{E}}{2\pi^2}\left[\frac{2m}{\hbar^2}\right]^{3/2}$
2	$\frac{k}{\pi d_z}$	$\frac{m}{\pi\hbar^2 d_z}$
1	$\frac{2}{\pi d_x d_y}$	$\frac{\rho(k)}{\sqrt{E}}\left[\frac{2m}{\hbar^2}\right]^{1/2}$

For parabolic bands, $\rho_r(E_{21}) \leftrightarrow \rho_c(E_2),\ \rho_v(E_1)$

$E_{21} - E_g \leftrightarrow E_2 - E_c,\ E_v - E_1$

$m_r \leftrightarrow m_c,\ m_v$

$$H'_{21} \triangleq \langle \psi_2 | H'(\vec{r}) | \psi_1 \rangle = \int_V \psi_2^* \cdot H'(\vec{r}) \cdot \psi_1 \cdot d^3 r$$

$$H'_{21} = \frac{q}{2m_0} \cdot \int_V F_2^* \cdot u_c^* \left(A(\vec{r}) \cdot \hat{e} \cdot \vec{p} \right) F_1 \cdot u_v \, d^3 r =$$

$$\left(\vec{p} \cdot A \cdot B = B \cdot \vec{p} \cdot A + A \, \vec{p} \cdot B \right)$$

$$= \frac{q}{2m_0} \cdot \left[\int_V u_c^* \cdot u_v \, \underbrace{F_2^* (A(\vec{r}) \hat{e} \cdot \vec{p}) \cdot F_1}_{\mathrm{I}} \cdot d^3 r \; + \right.$$

$$\left. \int_V \left[F_2^* A(\vec{r}) \cdot F_1 \right] \cdot \underbrace{u_c^* \hat{e} \cdot \vec{p} \cdot u_v}_{\mathrm{II}} \, d^3 r \right]$$

I cancels, because $\int_V \longrightarrow \sum_i \int_{\text{unit cell}}$

slowly-varying functions get out of $\int_{\text{unit cell}}$

and $\langle u_c | u_v \rangle_{\text{unit cell}} = 0$

$$H'_{21} = \frac{q}{2m_0} \sum_j \left[F_2^* A(\vec{r}) \cdot F_1 \right]_{\vec{r} = \vec{r}_j} \cdot \int_{\text{unit cell}} u_c^* \, \hat{e} \cdot \vec{p} \cdot u_v \cdot d^3 r =$$

$$= \frac{q}{2m_0} \langle u_c | \hat{e} \cdot \vec{p} | u_v \rangle \cdot \int_V F_2^* A(r) \cdot F_1 \, d^3 r$$

$$A(\vec{r}) = A_o \cdot e^{-i\vec{k}\vec{r}}$$ (variation much slower than the enveloppe function)

$$\int_V F_2^* A(r) \cdot F_1 \, d^3r \simeq A_o \cdot \int_V F_2^* F_1 \, d^3r = A_o \langle F_2 | F_1 \rangle$$

Then $$\boxed{|H'_{21}|^2 = \left(\frac{q \cdot A_o}{2m_o}\right)^2 \cdot |M_T|^2}$$

where $$\boxed{|M_T|^2 \triangleq \underbrace{|\langle u_c | \hat{e} \cdot \vec{p} | u_v \rangle|^2}_{|M|^2} |\langle F_2 | F_1 \rangle|^2}$$

contains polarisation information
$\simeq$ constant for every material
and given symmetry of wavefunction

$|M|^2$

	$\frac{2|M|^2}{m_o}$ in eV
GaAs $\rightarrow$	29
$Al_xGa_{1-x}As$	$29 + 2.85x$

$$\langle F_2 | F_1 \rangle = \frac{1}{V} \cdot \int_V e^{i\vec{k}_2 \cdot \vec{r}} e^{-i\vec{k}_1 \vec{r}} d^3r = \delta_{\vec{k}_1, \vec{k}_2}$$ for the bulk

TABLE 4.2 Magnitude of $|M_T|^2/|M|^2$ for Different Transitions and Polarizations.

	Bulk		Quantum-well ($k_t \sim 0$)	
Polarization	C–HH	C–LH	C–HH	C–LH
TE	1/3	1/3	1/2	1/6
TM	1/3	1/3	0	2/3

QW case:

$$\langle F_2 | F_1 \rangle = \frac{1}{A} \cdot \int_V F_2^*(z)\, e^{i\vec{k}_2 \cdot \vec{r}_{\parallel}} \cdot F_1(z) \cdot e^{-i\vec{k}_1 \vec{r}_{\parallel}}\, d\vec{r}^{\,3}$$

$$= \delta_{\vec{k}_2, \vec{k}_1} \cdot \int_z F_2^*(z) \cdot F_1(z)\, dz$$

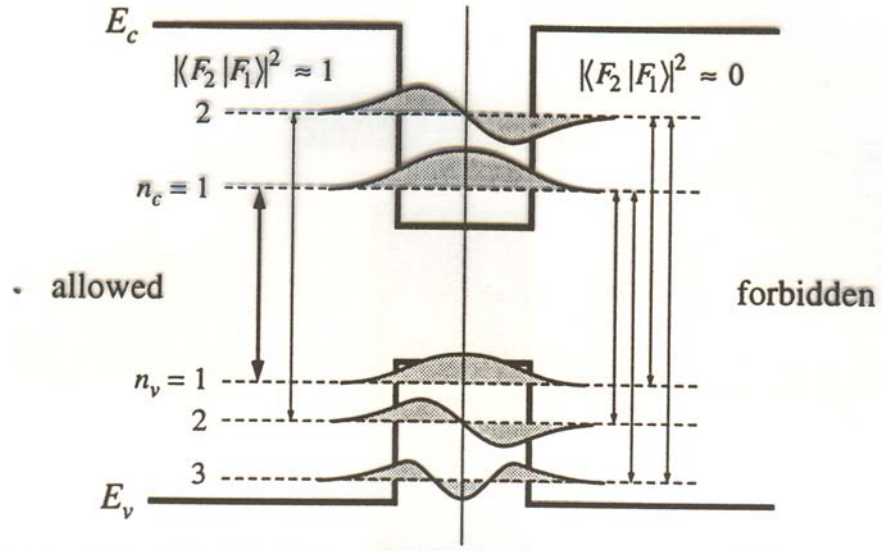

FIGURE 4.4 Allowed and forbidden transitions in a quantum well. The most important "$n = 1$" transition is highlighted in bold.

$$|\langle F_2 | F_1 \rangle|^2 \simeq \delta_{n_c, n_v}$$

Optical gain

$$g = \frac{1}{N_p} \cdot \frac{dN_p}{dt} = \frac{1}{v_g \cdot N_p} \cdot \frac{dN_p}{dt} = \frac{1}{v_g \cdot N_p} \cdot (R_{21} - R_{12}) =$$

$$= \frac{R_r}{v_g \cdot N_p} \cdot (f_2 - f_1) \Rightarrow$$

$$\boxed{g_{21} = \frac{2\pi}{\hbar} \cdot \frac{|H'_{21}|^2}{v_g \cdot N_p} \cdot \rho_r(E_{21}) \cdot (f_2 - f_1)}$$

(photon energy density)

$$\hbar\omega \cdot N_p = \frac{1}{2} n \cdot n_g \, \varepsilon_0 \cdot |\mathcal{E}|^2 = \frac{1}{2} \cdot n n_g \cdot \epsilon_0 \cdot \omega^2 |A_0|^2$$

dispersive medium

$$\frac{|H'_{21}|^2}{N_p} = \frac{1}{N_p} \cdot \left(\frac{q A_0}{2m_0}\right)^2 \cdot |M_T|^2 = \frac{q^2 \hbar}{2n \cdot n_g \cdot \epsilon_0 \cdot m_0^2 \omega} \cdot |M_T|^2$$

Then,

$$g_{21} = g_{max}(E_{21}) \cdot (f_2 - f_1)$$

$$g_{max}(E_{21}) = \frac{\pi \cdot q^2 \hbar}{n \epsilon_0 \cdot c m_0^2} \cdot \frac{1}{E_{21}} \cdot |M_T(E_{21})|^2 \cdot \rho_r(E_{21})$$

QWs

$$\boxed{g_{21} = \sum_{n_c} \cdot \sum_{n_v} g_{21}^{sub}(n_c, n_v)}$$

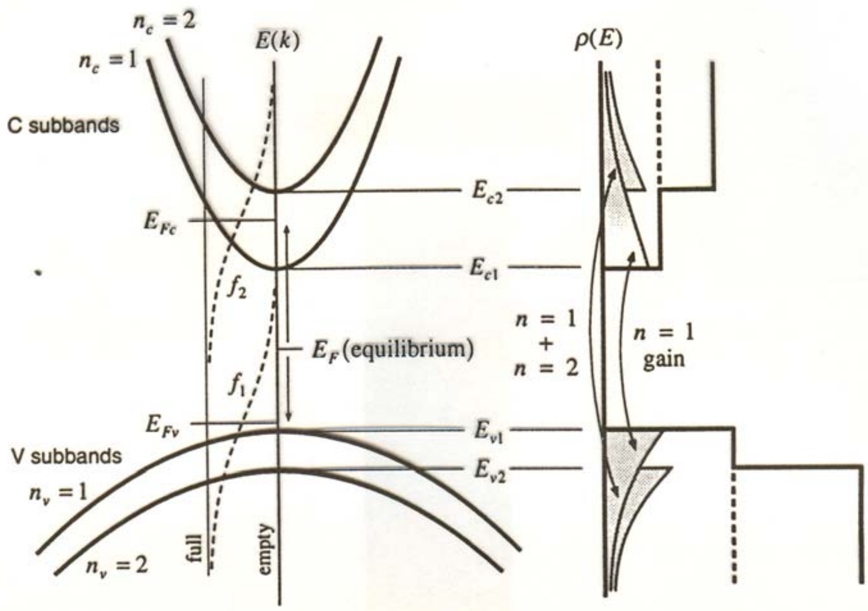

FIGURE 4.6 QW subbands and corresponding density of states illustrating the relationships between the carrier populations, the quasi-Fermi levels, and the gain at the subband edges.

- usually assume $n = p$
- since density of states in VB larger than CB E_{Fv} can be above E_v and still have gain ($E_{Fc} - E_{Fv} > E_g$)

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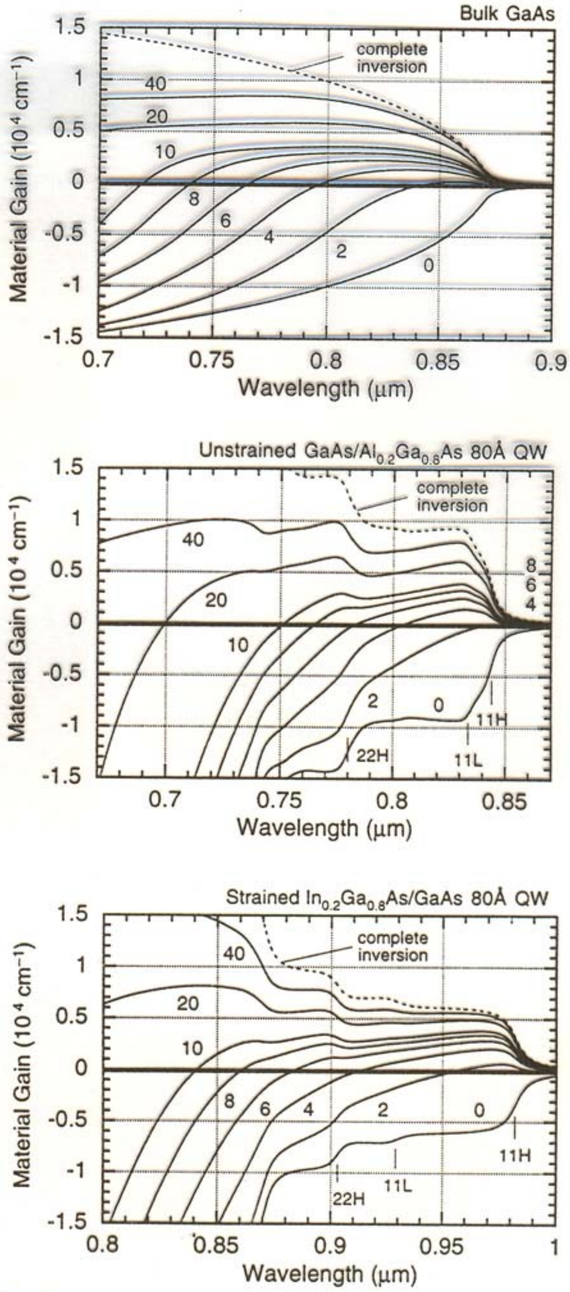

FIGURE 4.18 TE gain spectrum vs. carrier density in GaAs based materials. Indicated values are the sheet carrier densities: $\times 10^{12}$ cm^{-2} (the bulk "sheet" density assumes an 80Å width).