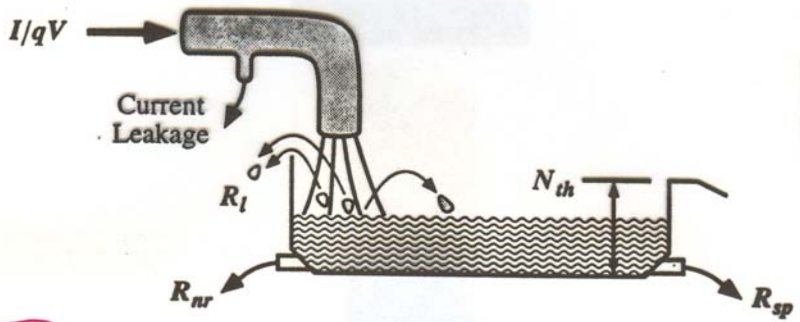
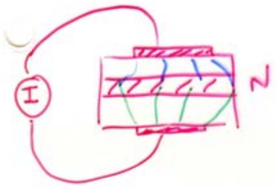
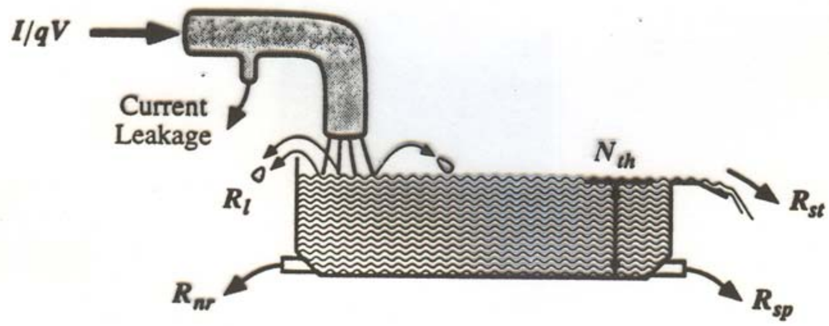
Σχηματική λειτουργία διοδ. λέιζερ

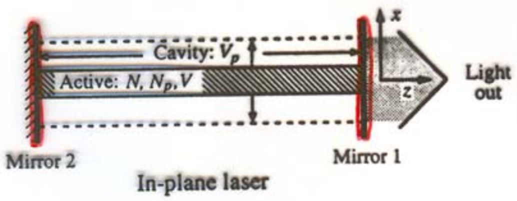




$$\frac{dN}{dt} = G_{gen} - R_{rec} =$$

$$= \frac{\eta_i I}{qV} - \left(R_{sp} + R_{nr} + R_l + R_{st} \right)$$





In-plane laser

$$\boxed{\frac{dN}{dt} = \frac{\eta_i I}{q \cdot V} - R_{Rec}}$$

$$R_{rec} = R_{sp} + (R_{nr} + R_e) + R_{st}$$

$$= B' N^2 + (AN + CN^3) + R_{st} =$$

$$= N \cdot (B \cdot N + A + CN^2) + R_{st}$$

$$= \frac{N}{\tau_{(N)}} + R_{st}$$

$$\boxed{\frac{dN_p}{dt} = \Gamma \cdot R_{st} + \Gamma \beta_{sp} \cdot R_{sp} - \frac{N_p}{\tau_p}}$$

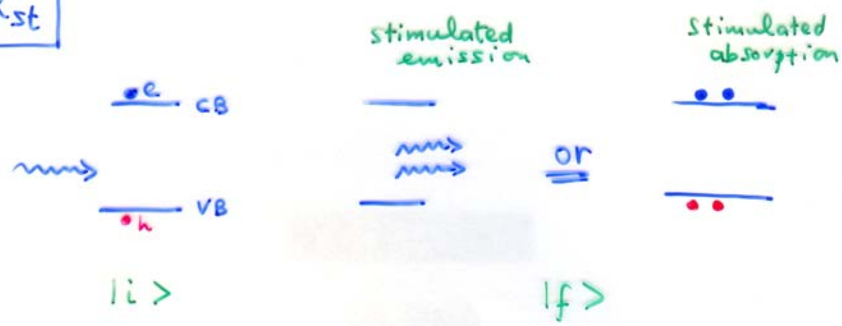
N_p = number of photons in a particular mode of the cavity

$$\Gamma = \frac{V}{V_p} = \Gamma_x \cdot \Gamma_y \cdot \Gamma_z = \frac{d}{d_{eff}} \cdot \frac{w}{w_{eff}} \cdot \frac{L_a}{L} \quad \text{(confinement factor)}$$

β_{sp} = fraction of SP photons coupled to a particular mode of the cavity.

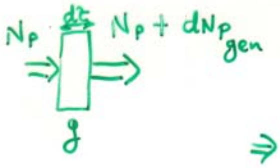
$\frac{N_p}{\tau_p}$ = losses of the cavity (scattering, residual absorption in the waveguide, mirrors)

R_{st}



R_{st} = stimulated emission − absorption
E A

For high enough densities, $E > A$ and photon population is amplified.



$$dN_{p_{gen}} = N_p g \cdot dz = N_p g \cdot v_g \cdot dt \longrightarrow \frac{\partial \omega}{\partial p}$$

$$\Rightarrow \left(\frac{dN_p}{dt}\right)_{gen} = R_{st} = v_g \cdot g \cdot N_p$$

$$\boxed{\begin{aligned} \frac{dN}{dt} &= \frac{\eta_i I}{q \cdot V} - \frac{N}{\tau_{(N)}} - v_g \cdot g \cdot N_p \\ \frac{dN_p}{dt} &= \Gamma\, v_g \cdot g \cdot N_p + \Gamma\, \beta_{sp} \cdot R_{sp} - \frac{N_p}{\tau_p} \end{aligned}}$$

A reasonable approximation near threshold $\Big\}\rightarrow$ $g \simeq a \cdot (N - N_{tr})$ where $a = \frac{\partial g}{\partial N} \triangleq$ differential gain

$\hookrightarrow$ transparency carrier density

if $g > 0 \rightarrow$ amplification

Σχόλιο για κυματοδηγούς με gain ή απορρόφηση

$\boxed{\tilde{n} = n + i\kappa}$, όπου $\kappa = \alpha \cdot \dfrac{\hbar c}{\hbar \omega}$

συντελεστής απορρόφησης (cm^{-1})

Τις περισσότερες φορές, $\kappa \ll n$. Για παράδειγμα,

$$\frac{\hbar c}{\hbar \omega} = \frac{0{,}67\ meV \cdot ps \cdot 3 \cdot 10^{8}\ m/s}{3\ eV} = 0{,}67 \cdot 10^{-5}\ cm$$

Δεδομένου ότι α κυμαίνεται μεταξύ $10 - 10^{4}\ cm^{-1}$

$\Rightarrow \quad \kappa \ll 1$ και επομένως $\boxed{\kappa \ll n}$.

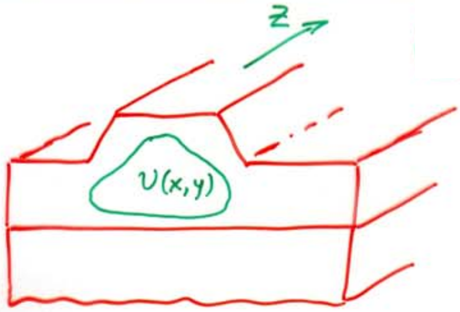
Επομένως, οι λύσεις κυματοδηγού (β_m) που βρήκαμε για $\kappa = 0$, ισχύουν προσεγγιστικά και όταν $\kappa \neq 0$, [redacted] αρκεί κ να παραμένει μικρό σε σχέση με το n $(\kappa \ll n)$.

Τότε η σταθερά μετάδοσης θα είναι της μορφής

$$\boxed{\beta_m + i\,\beta_{im}}$$

όπου β_m οι λύσεις για $\kappa = 0$, και β_{im} περιέχουν τους συντελεστές gain (g) και εσωτερικών απωλειών (α_i).

Threshold condition



$$\vec{\mathcal{E}} = \underbrace{\hat{e}_y}_{TE} \cdot E_0 \cdot U(x,y) \cdot e^{i(\omega t - \tilde{\beta} \cdot z)}$$

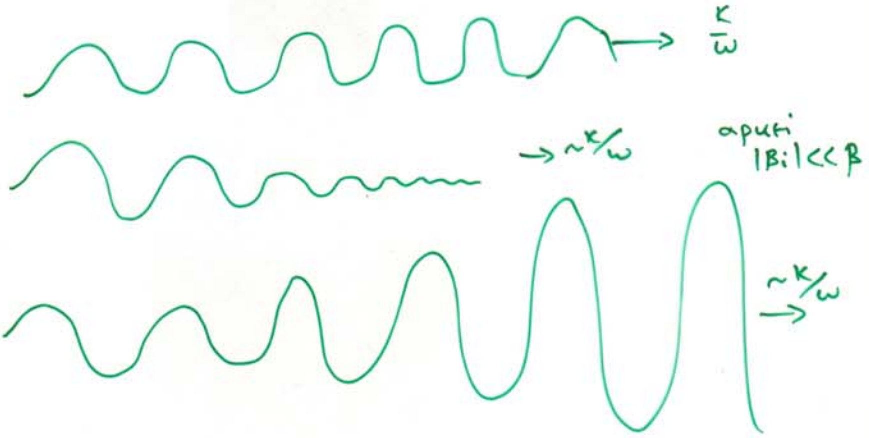
$$\tilde{\beta} = \underbrace{\beta}_{\frac{2\pi \bar{n}}{\lambda}} + i\beta_i = \beta + \frac{i}{2}\left(\underbrace{\Gamma_{xy} \cdot g}_{\text{modal gain}} - \underbrace{\alpha_i}_{\text{cavity losses}} \right)$$

where $$\Gamma_{xy} = \frac{\int_{-d/2}^{+d/2}\int_{-w/2}^{+w/2} |U(x,y)|^2 \cdot dx \cdot dy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |U(x,y)|^2 \cdot dx dy}$$

$\beta_i = 0$

$\beta_i < 0$

$\beta_i > 0$



Condition for lasing: non-zero transmitted wave for zero incident wave.

$z=0$ out $\quad \vec{\mathcal{E}}_{inc} = \hat{e}_y \cdot E_0 \cdot U \cdot e^{i(\omega t - \tilde{\beta} \cdot z)} = \vec{A}$

$z=0$ in $\quad \vec{\mathcal{E}}(z=0^+) = \vec{A} \cdot t_1$

$z=L$ in $\quad \vec{\mathcal{E}}(z=L^-) = \vec{A} \cdot t_1 \cdot e^{-i\tilde{\beta} \cdot L}$

$z=L$ out $\quad \vec{\mathcal{E}}(z=L^+) = \vec{A} \cdot t_1 \cdot t_2 \cdot e^{-i\tilde{\beta} \cdot L}$

$$\vec{\mathcal{E}}_{tr} = \vec{A} \cdot t_1 t_2 \cdot e^{-i\tilde{\beta} \cdot L} +$$

$$+ \vec{A} \cdot t_1 t_2 \cdot e^{-i\tilde{\beta} \cdot L} \left(\underbrace{r_2 \cdot r_1 \cdot e^{-i 2\tilde{\beta} \cdot L}}_{\text{one round trip}} \right) + \ldots$$

$$+ \vec{A} \cdot t_1 t_2 \cdot e^{-i\tilde{\beta} \cdot L} \cdot \left(\underbrace{r_2 \cdot r_1 \cdot e^{-i 2\tilde{\beta} \cdot L}}_{m \text{ round trips}} \right)^m + \ldots \quad = \text{γεωμ. πρόοδ}$$

$$= \frac{\vec{A} \cdot t_1 t_2 \cdot e^{-i\tilde{\beta} \cdot L}}{1 - r_1 r_2 \cdot e^{-i 2\tilde{\beta} \cdot L}}$$

Θέλω $\vec{\mathcal{E}}_{tr} \neq 0$ για $\vec{A} \to 0$. Με άλλα λόγια η ικανότητα laser να γεννά φωτόνια. $\Longleftrightarrow$

$$\boxed{1 - r_1 r_2 \cdot e^{-i 2\tilde{\beta} \cdot L} = 0}$$

ή σωστότερα τείνει στο μηδέν

Denominator diverges when

$$\boxed{e^{i\,2L\cdot\beta} = 1} \Rightarrow \underset{\uparrow \atop \frac{2\pi\bar{n}}{\lambda}}{L\cdot\beta} = m\pi \Rightarrow \lambda_m = \frac{2\bar{n}\cdot L}{m}$$

longitudinal cavity modes

$\bar{n} = \bar{n}(\lambda)$

$$\delta\lambda = \frac{\lambda^2}{2\cdot L\left(\bar{n} - \lambda\cdot\frac{\partial\bar{n}}{\partial\lambda}\right)}$$

(βλ. τελευταία διαφάνεια)

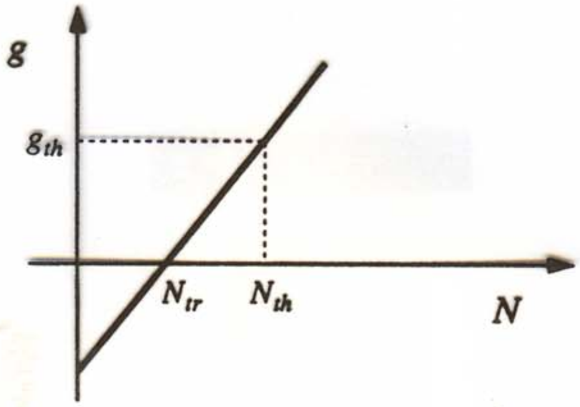
$$\boxed{1 = \underbrace{r_1 r_2}_{(R_1 R_2)^{1/2}} e^{(\Gamma_{xy}\cdot g - \alpha_i)L}} \Rightarrow \boxed{\Gamma_{xy}\cdot g_{threshold} = \alpha_i + \frac{1}{L}\cdot \ln\left(\frac{1}{R}\right)}$$

R

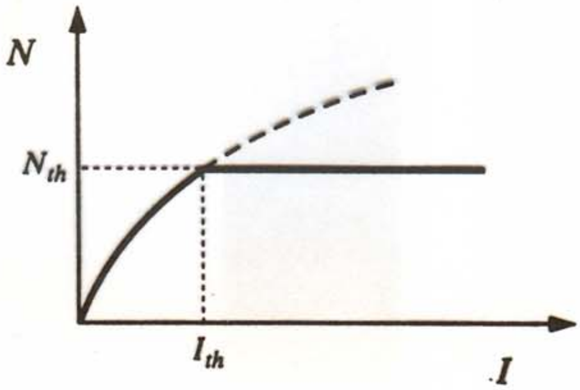
$$\boxed{\begin{aligned} g\,(I > I_{th}) &= g_{th} \\ N\,(I > I_{th}) &= N_{th} \end{aligned}}$$

(steady state)

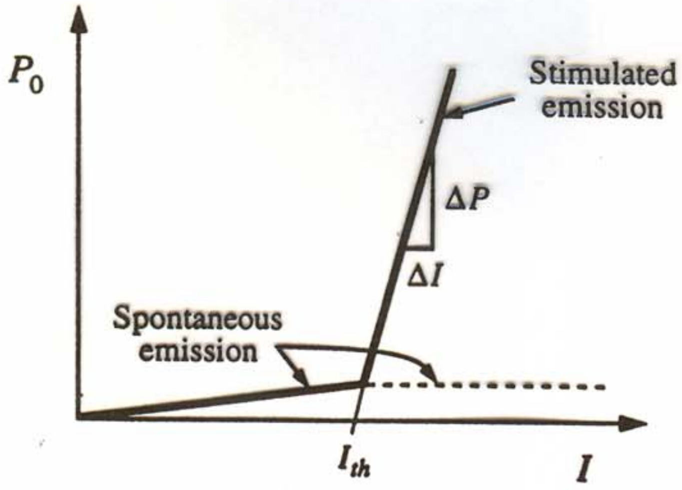
Dynamically, if $I > I_{th}$ then N, g increase so does N_p, R_{st} increases which balances N and g until new equilibrium is reached where $g = g_{th}$, $N = N_{th}$



g
g_{th}
N_{tr}
N_{th}
N



N
N_{th}
I_{th}
I



P_0
Stimulated emission
ΔP
ΔI
Spontaneous emission
I_{th}
I

$$\frac{dN}{dt} = \frac{n_i I}{q \cdot V} - \frac{N}{\tau} - v_g \cdot g \cdot N_p$$

$$\frac{dN_p}{dt} = \Gamma v_g \, g \cdot N_p + \Gamma \beta_{sp} \cdot R_{sp} - \frac{N_p}{\tau_p}$$

just below threshold $R_{st} \simeq 0$, steady state

$$\boxed{\frac{n_i \, I_{th}}{q \cdot V} = \frac{N_{th}}{\tau_{(N_{th})}}}$$

(Above threshold
$N = N_{th}$
$g = g_{th}$)

Above threshold, steady state

$$\frac{dN}{dt} = 0 = n_i \cdot \frac{(I - I_{th})}{q \cdot V} - v_g \cdot g_{th} \cdot N_p \Rightarrow$$

$$N_p = \frac{n_i \cdot (I - I_{th})}{q \cdot v_g \cdot g_{th} \cdot V}$$

$$P \propto N_p \propto I - I_{th}$$

slope: $$\boxed{P = n_i \left(\frac{\alpha_m}{\langle a_i \rangle + \alpha_m} \right) \frac{h\nu}{q} \cdot (I - I_{th})}$$

όπου $\alpha_m = \frac{1}{2L} \cdot \ln \frac{1}{R_1 R_2}$

$\langle \alpha_i \rangle$ = cavity losses

$$\boxed{\begin{aligned}\frac{dN}{dt} &= \frac{\eta_i I}{qV} - R_{sp} - v_g \cdot g \cdot N_p \quad (1)\\ \frac{dN_p}{dt} &= \Gamma \cdot v_g \cdot g \cdot N_p + \Gamma \cdot \beta_{sp} \cdot R_{sp} - \frac{N_p}{\tau_p} \quad (2)\end{aligned}}$$

$$\frac{dN_p}{dt} = 0 \Rightarrow \quad N_p = \frac{\Gamma \cdot \beta_{sp} \cdot R_{sp}}{\frac{1}{\tau_p} - \Gamma v_g \cdot g} \quad \Rightarrow \quad \left(\text{using } \frac{1}{\tau_p} = \Gamma \cdot v_g \cdot g_{th}\right)$$

where
$\Gamma g_{th} = \alpha_i + \alpha_m$

$$\Rightarrow \quad N_p = \frac{\beta_{sp} \cdot R_{sp}}{v_g \cdot (g_{th} - g)}$$

$$\Rightarrow \quad R_{st} = v_g \cdot g \cdot N_p = \frac{\beta_{sp} \cdot g \, R_{sp}}{g_{th} - g} \quad \Rightarrow$$

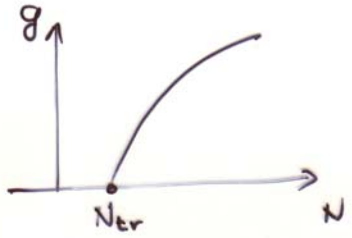
$$\boxed{\frac{R_{st}}{R_{sp}} = \frac{g}{g_{th} - g} \cdot \beta_{sp}}$$

Considering that β_{sp} is very small (10^{-5}), in order for R_{st} to be comparable to R_{sp}, Δg has to become very small $(\Delta g = 10^{-5} g)$.

Therefore, below threshold, it is reasonable to make the assumption that $\boxed{R_{st} \ll R_{sp}}$.

Ρεύμα κατωφλίου

$$g = g_0 \cdot \ln \frac{N}{N_{tr}}$$

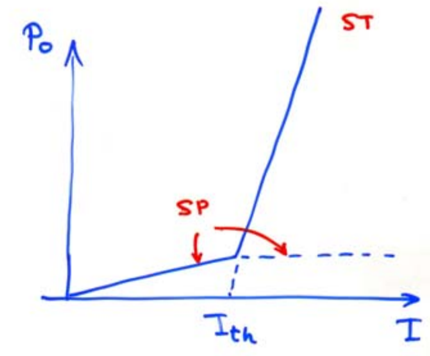


$$N_{th} = N_{tr} \cdot e^{g_{th}/g_0} =$$
$$= N_{tr} \cdot e^{(\langle\alpha_i\rangle + \alpha_m)/\Gamma \cdot g_0}$$

$$\boxed{I_{th} \simeq \frac{B \cdot N_{th}^2 \cdot q \cdot V}{n_i} = \frac{q \cdot V \cdot B \cdot N_{tr}^2}{n_i} \cdot e^{2(\langle\alpha_i\rangle + \alpha_m)/\Gamma \cdot g_0}}$$

$I_{th} \searrow$ όταν $\left\{ \begin{array}{l} \langle\alpha_i\rangle \searrow \\ \alpha_m \searrow \quad \text{(δηλαδή } L \nearrow, R \nearrow) \\ \Gamma \nearrow \\ V \searrow \end{array} \right.$

↗ παράμετροι που μπορούν να ελεγχθούν πειραματικά.



$$\boxed{P_o = \eta_i \, \frac{\alpha_m}{\alpha_i + \alpha_m} \cdot \frac{h\nu}{q} \cdot (I - I_{th})}$$

$$\frac{\eta_i \cdot \alpha_m}{\alpha_i + \alpha_m} = \eta_d \triangleq \text{differential efficiency} \longrightarrow \text{the larger the better}$$

When $L \to \infty$ or $R \to 1 \iff \alpha_m \to 0$, $\eta_d \to 0$

(you get zero photons out)

Similarly, when $\alpha_i \to \infty \Rightarrow \eta_d \to 0$.

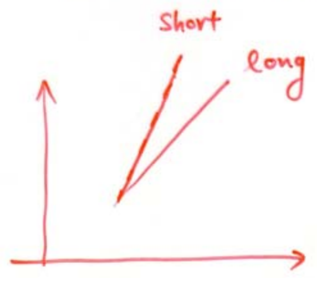
On the other hand, if $\alpha_i \to 0 \Rightarrow \eta_d = \eta_i$ (maximum efficiency)

$$\eta_d = \frac{\eta_i}{\frac{\alpha_i}{\alpha_m} + 1}$$

when $\frac{\alpha_i}{\alpha_m} \nearrow$ $\quad \eta_d \searrow$ $\quad \left(\overset{\text{MIN}}{0}\right)$

$\frac{\alpha_i}{\alpha_m} \searrow$ $\quad \eta_d \nearrow$ $\quad \left(\overset{\text{MAX}}{\eta_i}\right)$

If $L \searrow \Rightarrow \alpha_m \nearrow \Rightarrow \frac{\alpha_i}{\alpha_m} \searrow \Rightarrow \eta_d \uparrow$



How to measure $\langle a_i \rangle$, η_i

Take two different cavity lengths of same material.

Measure η_{d_1} and η_{d_2}

$$\left.\begin{aligned}
\eta_{d_1} &= \frac{\eta_i}{\frac{\langle a_i \rangle}{a_{m_1}} + 1} \\
\eta_{d_2} &= \frac{\eta_i}{\frac{\langle a_i \rangle}{a_{m_2}} + 1} \\
a_m &= \frac{1}{L} \cdot \ln \frac{1}{R}
\end{aligned}\right\} \Rightarrow \left\{\begin{matrix} \eta_i \\ \langle a_i \rangle \end{matrix}\right\}$$

$$\boxed{\begin{aligned}\Gamma \cdot g_{th} &= \langle a_i \rangle + \alpha_m = \\ &= \langle a_i \rangle + \frac{1}{L} \cdot \ln \frac{1}{R}\end{aligned}}$$

$$\text{if} \quad \left.\begin{aligned} L &\to \infty \\ R &\to 1 \end{aligned}\right\} \quad \Gamma \cdot g_{th} \to \langle a_i \rangle$$

$$\text{if} \quad R \to 0 \qquad \Gamma \cdot g_{th} \to \infty$$

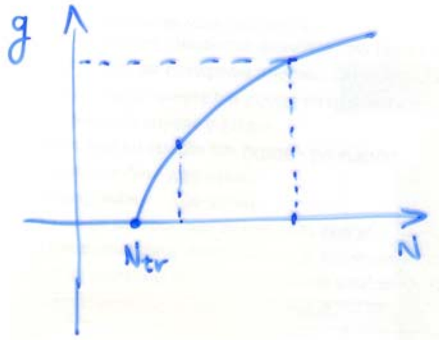
On the number of QWs in the active region.

Let $N_{QW} \to$ number of QWs

$$\Gamma_{N_{QW}} = N_{QW} \Gamma_1 \quad \xRightarrow{\langle a_i \rangle,\, a_m \text{ same}} \quad \boxed{g_N^{th} = \frac{1}{N_{QW}} \cdot g_1^{th}}$$

But

Assuming $g = g_0 \cdot \ln \frac{N}{N_{tr}}$



One can show that I_{th} in case of MQW is

$$I_{th}^{MQW} \cong \frac{q \cdot N_{QW} \cdot V_1 \cdot B \cdot N_{tr}^2}{\eta_i} \cdot e^{2(\langle a_i \rangle + \alpha_m)/N_{QW}\Gamma_1 g_0}$$

Example: $\Gamma_1 \simeq 0.1$, $g_0 \simeq 100\,cm^{-1}$, $\langle a_i \rangle = 10\,cm^{-1}$

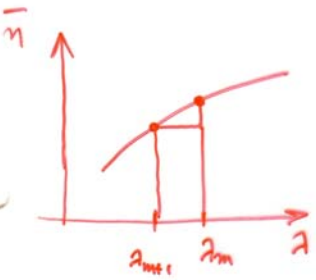
$R_1 = 1$, $R_2 = 0.32$, $L = 500\,\mu m$

$\Rightarrow$ $\boxed{\text{optimal number of QWs} \quad N_{QW} = 3}$

Wavelength difference between successive axial modes

$$\lambda_m = \frac{2}{m}\cdot \bar{n}(\lambda_m)\cdot L \quad (1) \quad \Rightarrow \quad m = \frac{2\bar{n}(\lambda_m)\cdot L}{\lambda_m} \quad (2)$$

$$(m+1)\cdot\lambda_{m+1} = 2\,\bar{n}(\lambda_{m+1})\cdot L \overset{(\beta\lambda.\,o\chi.)}{=} 2\cdot\left[\bar{n}(\lambda_m) - \frac{\partial \bar{n}}{\partial\lambda}(\lambda_m)\cdot\Delta\lambda\right]\cdot L \quad (3)$$



$\Delta\lambda \triangleq \lambda_m - \lambda_{m+1}$

Από (3)∧(1) ⇒

$$m\cdot\lambda_m - (m+1)\cdot\lambda_{m+1} = m\cdot\Delta\lambda - \lambda_{m+1} =$$

$$= 2\,\frac{\partial\bar{n}}{\partial\lambda}\cdot\Delta\lambda\cdot L \Rightarrow$$

$$m - \frac{\lambda_{m+1}}{\Delta\lambda} = 2\cdot\frac{\partial\bar{n}}{\partial\lambda}\cdot L \overset{(2)}{\Longrightarrow}$$

$$\Rightarrow \frac{\lambda_{m+1}}{\Delta\lambda} = 2L\cdot\frac{\bar{n}}{\lambda_m} - 2L\cdot\frac{\partial\bar{n}}{\partial\lambda} = \frac{2L}{\lambda_m}\cdot\left(\bar{n} - \lambda_m\cdot\frac{\partial\bar{n}}{\partial\lambda}\right) \Rightarrow$$

$$\frac{\lambda_m\cdot\lambda_{m+1}}{\Delta\lambda} \simeq \frac{\lambda^2}{\Delta\lambda} = 2L\underbrace{\left(\bar{n} - \lambda\cdot\frac{\partial\bar{n}}{\partial\lambda}\right)}_{\text{define } \bar{n}_g}$$

group refractive index

$$\Rightarrow \boxed{\Delta\lambda = \frac{\lambda^2}{2L\left(\bar{n} - \lambda\cdot\frac{\partial\bar{n}}{\partial\lambda}\right)} = \frac{\lambda^2}{2L\cdot\bar{n}_g}}$$