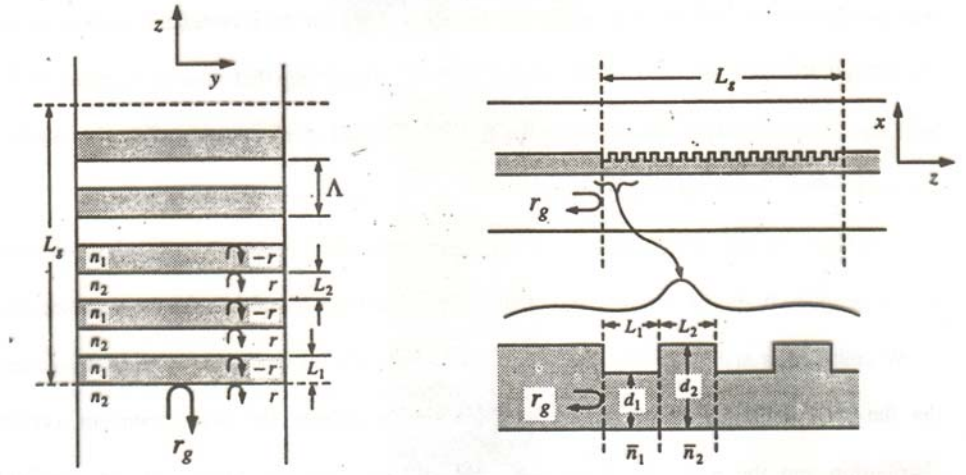
Gratings



VCSEL

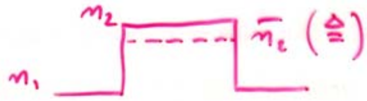
(a)

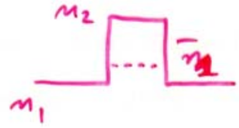
In-plane **DBR**

(b)

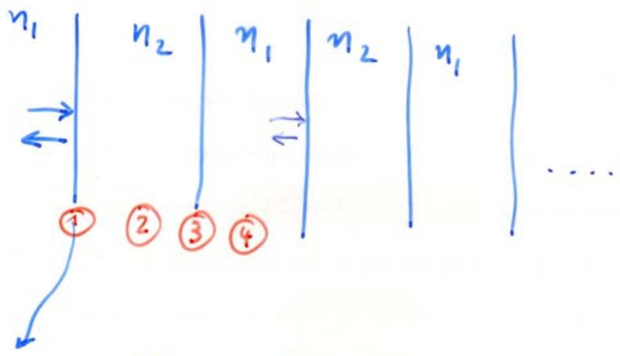
$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

$$\bar{r} = \frac{\bar{n}_1 - \bar{n}_2}{\bar{n}_1 + \bar{n}_2}$$





$\bar{n}_1 < \bar{n}_2$



1 $$T_{12} = \frac{1}{t_{12}} \cdot \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix}$$

2 $$D_2 = \begin{bmatrix} e^{-j\varphi_2} & 0 \\ 0 & e^{j\varphi_2} \end{bmatrix}$$

3 $$T_{21} = \frac{1}{t_{21}} \cdot \begin{bmatrix} 1 & -r_{12} \\ -r_{12} & 1 \end{bmatrix}$$

4 $$D_1 = \begin{bmatrix} e^{-j\varphi_1} & 0 \\ 0 & e^{j\varphi_1} \end{bmatrix}$$

$t_{12} = t_{21}$

$r_{12} = -r_{21}$

$\varphi_1 = \tilde{\beta}_1 \cdot L_1$

$\varphi_2 = \tilde{\beta}_2 \cdot L_2$

$$T = T_{12} \cdot D_2 \cdot T_{21} \cdot D_1 =$$

$$= \frac{1}{t^2} \cdot \begin{bmatrix} e^{-i\varphi_1}\left(e^{-i\varphi_2} - r^2 e^{i\varphi_2}\right) & 2i \sin\varphi_2 \cdot r \cdot e^{i\varphi_1} \\ -2i\sin\varphi_2 \cdot r \cdot e^{-i\varphi_1} & e^{i\varphi_1}\left(e^{i\varphi_2} - r^2 e^{-i\varphi_2}\right) \end{bmatrix}$$

if no loss $\Rightarrow$ $\varphi_1 = \frac{2\pi n_1}{\lambda} \cdot L_1$, $\varphi_2 = \frac{2\pi n_2}{\lambda} \cdot L_2$

Quarter-wave case

$$\left\{ \begin{array}{ll} L_1 = \dfrac{\lambda_B}{4n_1} & \rightarrow \quad \varphi_1 = \pi/2 \\ L_2 = \dfrac{\lambda_B}{4n_2} & \rightarrow \quad \varphi_2 = \pi/2 \end{array} \right\}$$

$$\boxed{T = -\frac{1}{t^2}\cdot\begin{bmatrix} 1+r^2 & 2r \\ 2r & 1+r^2 \end{bmatrix}}$$

Eigenvalues $a_+ = (1+r)^2$

$a_- = (1-r)^2$

Eigenvectors $V_+ = \frac{1}{\sqrt{2}}\cdot\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$V_- = \frac{1}{2}\cdot\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$T = A^{-1}\cdot M\cdot A$ where $M = \frac{1}{t^4}\cdot\begin{bmatrix} (1+r)^2 & 0 \\ 0 & (1-r)^2 \end{bmatrix}$

$A = \frac{1}{\sqrt{2}}\cdot\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, $A^{-1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Bragg mirror with N - quarter wave pairs $\Rightarrow$

$$T^N = \underbrace{(A^{-1}MA)(A^{-1}MA)\ldots}_{N \text{ times}} = A^{-1}M^N A =$$

$$= \frac{1}{t^{4N}}\cdot A^{-1}\cdot\begin{bmatrix} (1+r)^{2N} & 0 \\ 0 & (1-r)^{2N} \end{bmatrix} A \qquad \left(t = \sqrt{1-r^2}\right)$$

$$\left(R_N = \left| \frac{T_{21}^N}{T_{11}^N} \right|^2 \right)$$

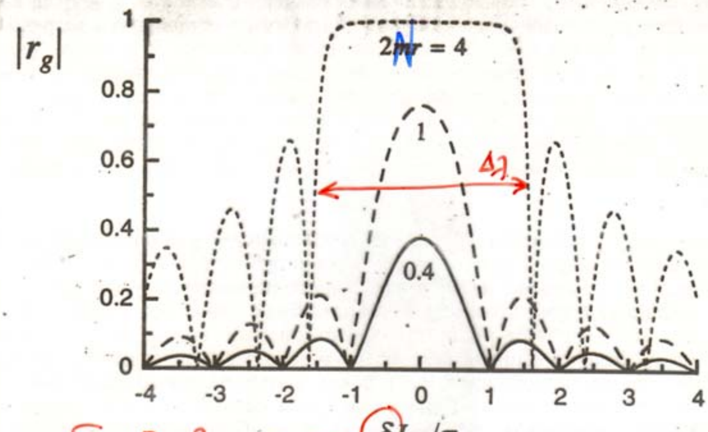
$$\Rightarrow \quad R_N = \left| \frac{(1+r)^{2N} - (1-r)^{2N}}{(1+r)^{2N} + (1-\rho)^{2N}} \right|^2 \quad \Rightarrow \quad \left(r = \frac{n_1 - n_2}{n_1 + n_2} > 0 \right)$$

$$\boxed{R_N = \left[\frac{1 - \left(\frac{n_2}{n_1}\right)^{2N}}{1 + \left(\frac{n_2}{n_1}\right)^{2N}} \right]^2} \quad \text{(at Bragg wavelength)}$$

, where $\frac{n_2}{n_1} < 1$

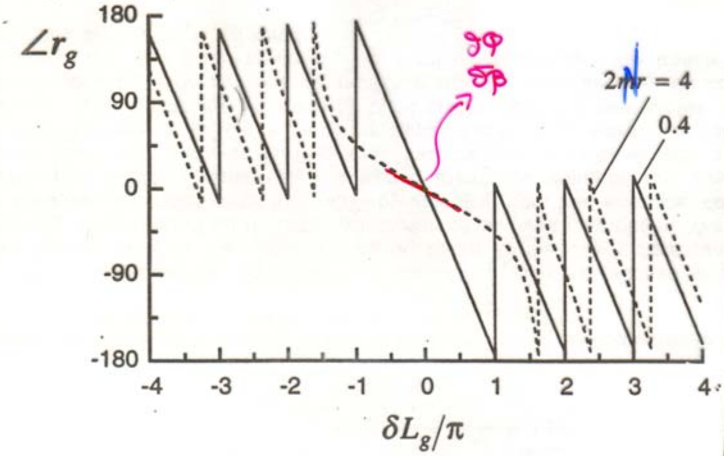
For $N \to \infty$ $\quad R_N \to 1$

For $\lambda \neq \lambda_{Bragg}$ $\to$ use computers



$\delta = \beta - \beta_0$

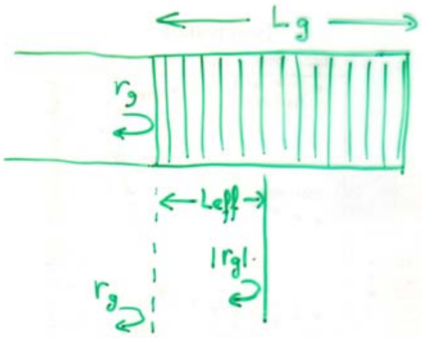
(a)



(b)

$$\frac{\Delta\lambda}{\lambda_g} = \frac{4}{\pi}\cdot\left[\frac{n_1-n_2}{n_1+n_2}\cdot \ln\frac{n_1}{n_2}\right]^{1/2}$$

$$\ln\frac{n_1}{n_2} \xrightarrow{n_1 \simeq n_2} \frac{n_1-n_2}{n_2} \; ; \quad \boxed{\frac{\Delta\lambda}{\lambda_B} \simeq \frac{4}{\pi\sqrt{2}}\cdot\frac{\Delta n}{n}}$$



$r_g = |r_g| e^{i\varphi_{eff}}$

In the discussion of succesive modes in the cavity,

$$\boxed{L_{eff} = -\frac{1}{2} \cdot \frac{\partial \varphi_{eff}}{\partial \beta}}$$

when $2N \cdot r \nearrow$ $\quad L_{eff} \searrow$ (previous figure)

It can be shown that near Bragg-wavelength

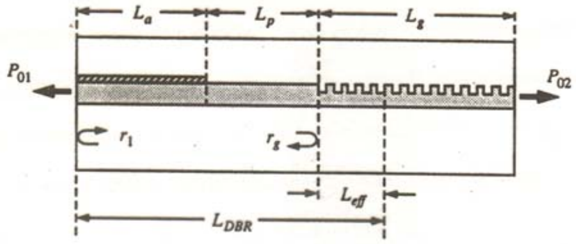
$$\boxed{L_{eff} = \frac{L_g}{4Nr} \cdot \tanh(2Nr)}$$

2Nr small $\quad \boxed{L_{eff} \to \frac{L_g}{2}}$

2Nr large $\quad \tanh \to 1 \quad \boxed{L_{eff} \to \frac{\Lambda}{4r}}$

Leff has also the meaning of field penetration into the mirror.

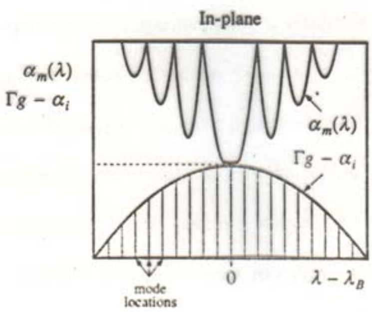
DBR lasers



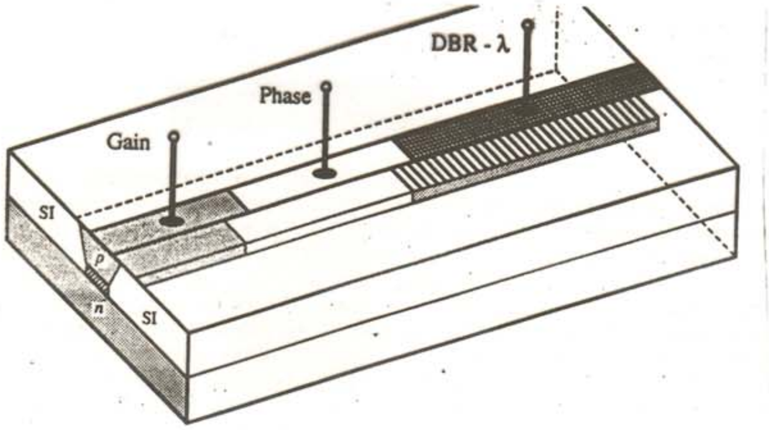
$$\Gamma \cdot g_{th} = \langle \alpha_i \rangle + \frac{1}{L_a + L_p} \cdot \ln \cdot \left[\frac{1}{r_1 |r_g|} \right]$$

modes: $$\Delta\lambda = \frac{\lambda^2}{2\left[\bar{n}_{ga} \cdot L_a + \bar{n}_{gp} \cdot L_p + \bar{n}_{g\,DBR} \cdot L_{eff}\right]}$$

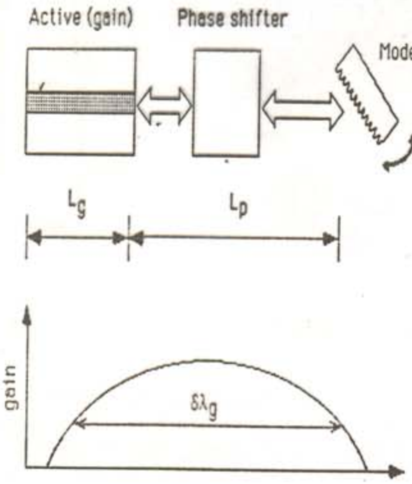
Mode selectivity

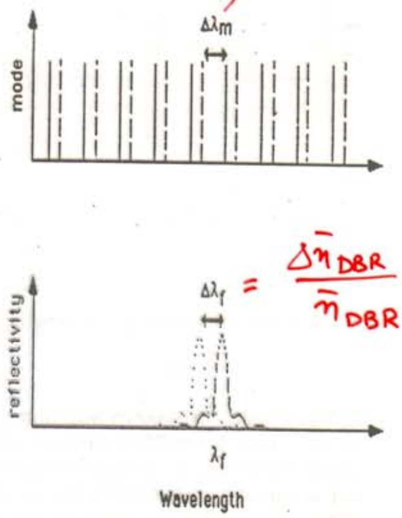


Tunability



$$\frac{\Delta\lambda_m}{\lambda_m} = \frac{\Delta\bar{n}_a \cdot L_a + \Delta\bar{n}_p \cdot L_p + \Delta\bar{n}_{DBR} \cdot L_{eff}}{\bar{n}_a \cdot L_a + \bar{n}_p \cdot L_p + \bar{n}_{DBR} \cdot L_{eff}}$$

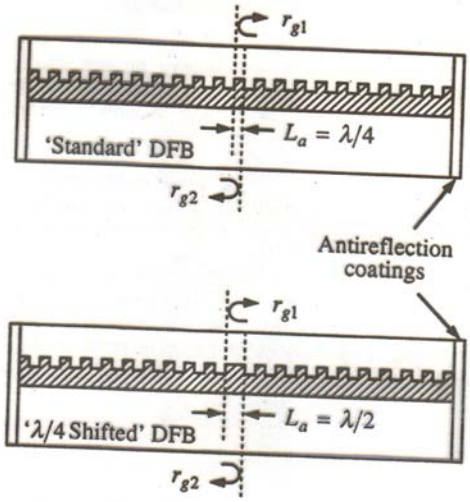




$$\Delta\lambda_f = \frac{\Delta\bar{n}_{DBR}}{\bar{n}_{DBR}} \cdot \lambda_f$$

$$\Delta\bar{n}(I) = \frac{\partial\bar{n}}{\partial N} \cdot \frac{\eta_i \cdot \tau \cdot I_j}{q \cdot V_j}$$

Distributed Feedback Laser



$$r_g = \frac{T_{21}}{T_{11}} \cdot n_{eff} \cdot \frac{1 + j\Delta}{1 + j\, n_{eff}\, \Delta}$$

At Bragg wavelength, λ_0,

$$\Delta \to 0$$

$$T_{21} \to -\frac{2r}{t^2}$$

$$T_{11} \to -\frac{1+r^2}{t^2}$$

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t = \frac{2\sqrt{n_1 n_2}}{n_1 + n_2}$$

Standard DFB

$$\boxed{r_{g_1}(\tilde{\beta}_{th}) \cdot r_{g_2}(\tilde{\beta}_{th}) \cdot e^{2i\tilde{\beta}_{th} \cdot \frac{\lambda}{4\bar{n}_2}} = 1}$$

$$\left(r_1 \cdot r_2 \cdot e^{2i\tilde{\beta} \cdot L} = 1 \right)$$

At Bragg, $\boxed{r_g \simeq \frac{T_{21}}{T_{11}} \cdot m_{eff}}$

Provided $\boxed{r = \text{real}} \Rightarrow \boxed{\frac{T_{21}}{T_{11}}}$ is real

$\Rightarrow \boxed{r_g = \text{real}}$

$$|r_{g_1}| \cdot e^{i\varphi_1} \cdot |r_{g_2}| \cdot e^{i\varphi_2} \cdot e^{2i\tilde{\beta}_{th} \cdot \frac{\lambda}{4\bar{n}_2}} = 1$$

phase condition $e^{i\left(\underset{0}{\overset{!!}{\varphi_1}} + \underset{0}{\overset{!!}{\varphi_2}} + \underbrace{2\left(\frac{2\pi}{\lambda} \cdot \bar{n}_2 \cdot \frac{\lambda}{4\bar{n}_2}\right)}_{\pi}\right)} = 1$

$\Rightarrow \quad -1 = 1$

Standard DFB, does not lase at Bragg wavelength but at two adjacent wavelengths

$\frac{\lambda}{4}$ -shifted DFB

$$e^{i(\varphi_1 + \varphi_2 + 2\pi)} = 1$$

for $\lambda = \lambda_0$, $\varphi_1 = \varphi_2 = 0$ ✓ lasing is passible at λ_0.

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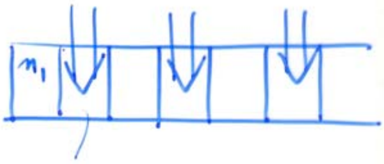
gain-coupled DFB

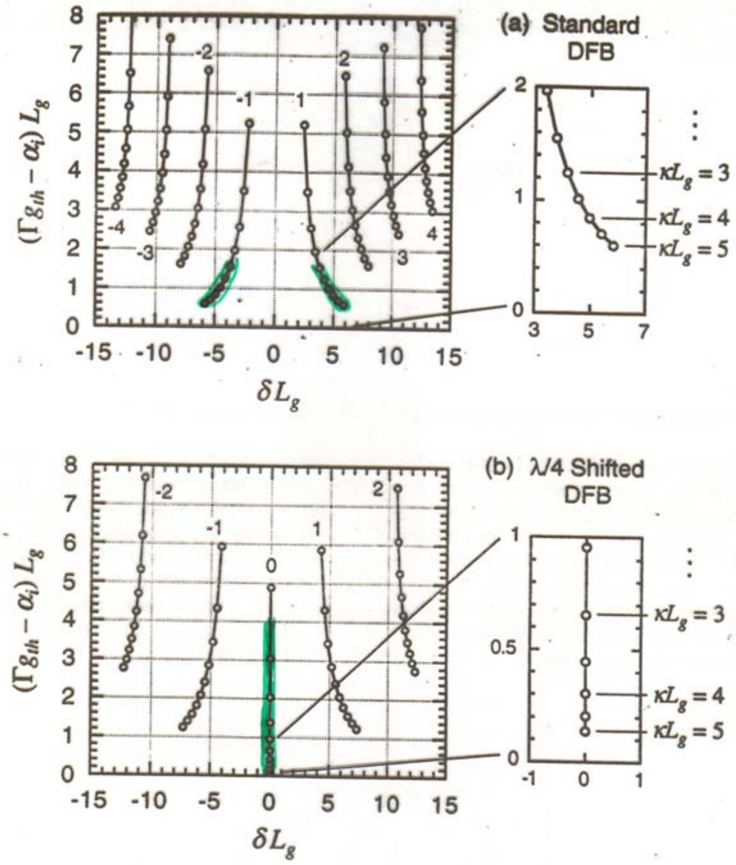


$$n_2 = n_1 + j \cdot n_i = n_1 + j \cdot \frac{g \cdot \lambda}{4\pi}$$

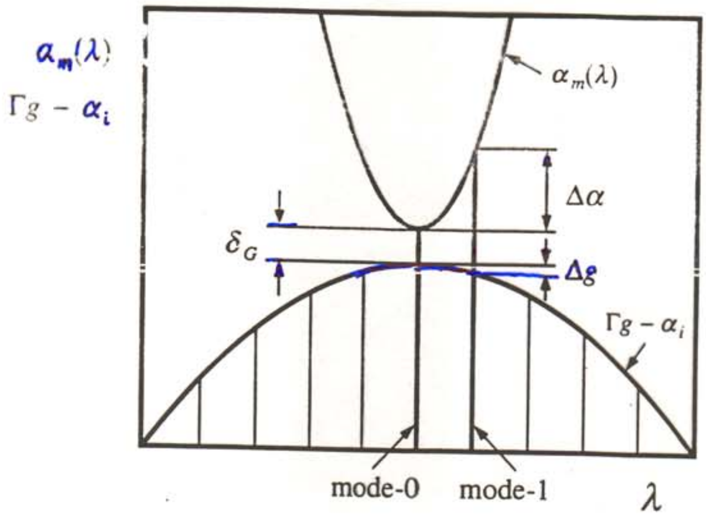
Then
$$r = \frac{n_2 - n_1}{n_2 + n_1} = \frac{j \cdot g\lambda/4\pi}{2n_1 + j\underbrace{g\lambda/4\pi}_{\text{small}}} \simeq j\,\frac{n_i}{2n_1} \simeq \text{imaginary}$$

$\Rightarrow$ r_g also imaginary. $\boxed{r_g = |r_g| \cdot e^{i\pi/2}}$

$$e^{i(\overset{\pi/2}{\varphi_1} + \overset{\pi/2}{\varphi_2} + \pi)} = 1 \Rightarrow 1 = 1$$


(a) Standard DFB
(b) λ/4 Shifted DFB
$(\Gamma g_{th} - \alpha_i) L_g$
δL_g
$\kappa L_g = 3$
$\kappa L_g = 4$
$\kappa L_g = 5$

Mode suppression ratio in single-mode lasers



$$MSR = \frac{P(\lambda_0)}{P(\lambda_1)}$$

$$P(\lambda_n) = \Gamma_1'(\lambda_n) \cdot v_g \cdot \alpha_m \cdot (\lambda_n) \cdot N_P(\lambda_n) \cdot h\nu \cdot V_P$$

$$N_p(\lambda_n) = \frac{\Gamma' \beta_{sp} \cdot R_{sp}(\lambda_n)}{1/\tau_p(\lambda_n) - \Gamma' v_g \cdot g(\lambda_n)}$$

$$\left(\frac{dN_p}{dt} = \Gamma v_g \cdot g \cdot N_p + \Gamma' \beta_{sp} \cdot R_{sp} - \frac{N_p}{\tau_p} \right) = 0$$

$$\frac{1}{\tau_p(\lambda_n)} = v_g \cdot (\alpha_i + \alpha_m)$$

For a large number of photons

$$\Gamma' v_g \cdot g(\lambda_n) \longrightarrow \frac{1}{\tau_p(\lambda_n)}$$ but never reaches

In other words,

$$(SP) + \Gamma' g \leq \alpha_i + \alpha_m$$

$$MSR = \frac{F_1(\lambda_0) \cdot \alpha_m(\lambda_0) \cdot \left[\alpha_i + \alpha_m(\lambda_1) - \Gamma' g \cdot (\lambda_1) \right]}{\underbrace{F_1(\lambda_1) \cdot \alpha_m(\lambda_1)}_{\simeq 1} \cdot \left[\alpha_i + \alpha_m(\lambda_0) - \Gamma' g(\lambda_0) \right]}$$

Define $\delta_G = \alpha_m(\lambda_0) - \left[\Gamma' g(\lambda_0) - \alpha_i \right]$

$$\Delta\alpha = \alpha_m(\lambda_1) - \alpha_m(\lambda_0)$$

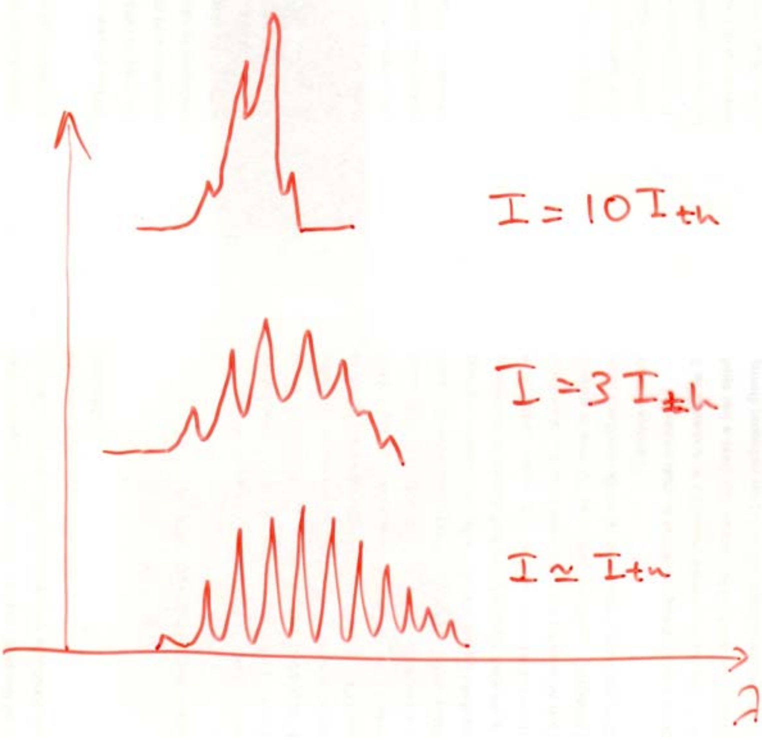
$$\Delta g = \Gamma' \cdot g(\lambda_0) - \Gamma' \cdot g(\lambda_1)$$

$$\boxed{MSR \simeq \frac{\Delta\alpha + \Delta g}{\delta_G} + 1}$$

$$\delta_G = \frac{F_1 \alpha_m \cdot h\nu \cdot V_p \cdot \Gamma \beta_{sp} \cdot R_{sp}}{P_{01}}$$

$\Rightarrow \quad I > I_{th}$, $$\boxed{\delta_G = (\alpha_i + \alpha_m) \cdot \beta_{sp} \cdot \eta_r \cdot \frac{I_{th}}{I - I_{th}}}$$

$I \nearrow \quad \delta_G \searrow \quad MSR \nearrow$



$\beta_{sp} \sim 10^{-4}$

$\alpha_i \sim 10 \text{ cm}^{-1}$

$\alpha_m \sim 10 \text{ cm}^{-1}$

$$\delta_G \simeq 10^{-3} \frac{I_{th}}{I - I_{th}} \text{ cm}^{-1}$$

Fabry-Perot laser $\qquad \Delta\alpha \simeq 0$

$$\Delta g = 0.5\,\%\,\underbrace{(\Gamma g_0 - \alpha_i)}_{10 \text{ cm}^{-1}}$$

$$MSR = \frac{0.5 \cdot 10 \cdot 10^{-2}}{10^{-3}} \simeq 50$$

DFB laser

$$\Delta\alpha = 10\,\%\ \alpha_m\ (1_\circ)$$

$$MSR = \frac{0.5\,\% \cdot 10 \text{ cm}^{-1} + 10\,\%\ 10 \text{ cm}^{-1}}{10^{-3}} \simeq 1000$$